Binary Representation

- The basis of all digital data is binary representation.
- · Binary means 'two'
 - 1,0
 - True, False
 - Hot, Cold
 - On, Off
- We must be able to handle more than just values for real world problems

V 0.1

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- 1,0,56
- True, False, Maybe
- Hot, Cold, LukeWarm, Cool
- On, Off, Leaky

Number Systems

- To talk about binary data, we must first talk about number systems
- The decimal number system (base 10) you should be familiar with!
 - A digit in base 10 ranges from 0 to 9.
 - A digit in base 2 ranges from 0 to 1 (binary number system). A digit in base 2 is also called a 'bit'.
 - A digit in base R can range from 0 to R-1
 - A digit in Base 16 can range from 0 to 16-1 (0,1,2,3,4,5,5,6,7,8,9,A,B,C,D,E,F). Use letters A-F to represent values 10 to 15. Base 16 is also called Hexadecimal or just 'Hex'.

Positional Notation

Value of number is determined by multiplying each digit by a weight and then summing. The weight of each digit is a POWER of the BASE and is determined by position.


```
 \% 1011.11 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2} 
= 8 + 0 + 2 + 1 + 0.5 + 0.25 
= 11.75
```

$$\begin{array}{rcl} \$ \ A2F &=& 10x16^2 + 2x16^1 + 15x16^0 \\ &=& 10x256 & +& 2x16 & +& 15x1 \\ &=& 2560 + 32 + 15 = \frac{2}{2}607 \\ &=& \frac{2}$$

Base 10, Base 2, Base 16
The textbook uses subscripts to represent different
bases (ie.
$$A2F_{16}$$
, 953.78_{10} , 1011.11_2)
I will use special symbols to represent the different bases.
The default base will be decimal, no special symbol for
base 10.
The '\$' will be used for base 16 (\$A2F)
Will also use 'h' at end of number (A2Fh)
The '%' will be used for base 2 (%10101111)
If ALL numbers on a page are the same base (ie, all in base
16 or base 2 or whatever) then no symbols will be used and
a statement will be present that will state the base (ie, all
numbers on this page are in base 16).

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	Common Powers	
$\begin{array}{l} 2^{-3}=0.125\\ 2^{-2}=0.25\\ 2^{-1}=0.5\\ 2^{0}=1\\ 2^{1}=2\\ 2^{2}=4\\ 2^{3}=8\\ 2^{4}=16\\ 2^{5}=32\\ 2^{6}=64\\ 2^{7}=128\\ 2^{8}=256\\ 2^{9}=512\\ 2^{10}=1024\\ 2^{11}=2048\\ 2^{12}=4096 \end{array}$	$16^{0} = 1 = 2^{0}$ $16^{1} = 16 = 2^{4}$ $16^{2} = 256 = 2^{8}$ $16^{3} = 4096 = 2^{12}$ $2^{10} = 1024 = 1 \text{ K}$ $2^{20} = 1048576 = 1 \text{ M (1 Megabits)} = 1024 \text{ K} = 2^{10} \text{ x } 2^{10}$ $2^{30} = 1073741824 = 1 \text{ G (1 Gigabits)}$	
	V 0.1	5





Divide Number N by base R until quotient is 0. Remainder at EACH step is a digit in base R, from Least Significant digit to Most significant digit. Convert 53 to binary

 $53/2 = 26, rem = 1 \leftarrow Least Significant Digit$ 26/2 = 13, rem = 013/2 = 6, rem = 16/2 = 3, rem = 03/2 = 1, rem = 1 $1/2 = 0, rem = 1 \leftarrow Most Significant Digit$ $<math display="block">53 = \% 110101 = 1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0 = 32 + 16 + 0 + 4 + 0 + 1 = 53$ 7



More Conversions Convert 53 to Hex 53/16 = 3, rem = 5 3/16 = 0, rem = 3 53 = 35h $= 3 \times 16^{1} + 5 \times 16^{0}$ = 48 + 5 = 53



Hex to Binary, Binary to Hex $A2Fh = \% 1010 \ 0010 \ 1111$ $345h = \% \ 0011 \ 0100 \ 0101$ Binary to Hex is just the opposite, create groups of 4 bits starting with least significant bits. If last group does not have 4 bits, then pad with zeros for unsigned numbers. $\% \ 1010001 = \% \ 0101 \ 0001 = 51h$ Padded with a zero

V 0.1

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Binary Numbers Again

Recall than N binary digits (N bits) can represent unsigned integers from 0 to 2^{N} -1.

4 bits = 0 to 15 8 bits = 0 to 255 16 bits = 0 to 65535

Besides simply representation, we would like to also do arithmetic operations on numbers in binary form. Principle operations are addition and subtraction.

V 0.1

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Binary Arithmetic, Subtraction

The rules for binary arithmetic are:

The rules for binary subtraction are:

0 + 0 = 0, carry = 0	
1 + 0 = 1, carry = 0	
0 + 1 = 1, carry = 0	
1 + 1 = 0, carry = 1	

0 - 0 = 0, borrow = 0 1 - 0 = 1, borrow = 0 0 - 1 = 1, borrow = 1 1 - 1 = 0, borrow = 0

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Borrows, Carries from digits to left of current of digit.

Binary subtraction, addition works just the same as decimal addition, subtraction.

Binary, Decimal addition Binary Decimal % 101011 34 + % 000001 + 17 ------101100 -----From LSB to MSB: 51 1+1 = 0, carry of 1 from LSD to MSD: 1 (carry) + 1 + 0 = 0, carry of 17+4 = 1; with carry out of 1 to next column 1 (carry) + 0 + 0 = 1, no carry 1 + 0 = 10 + 0 = 01 (carry) + 3 + 1 = 5.1 + 0 = 1answer = 51. answer = % 101100 V 0 1 15

Subtraction			
Decimal	Binary		
900	% 100		
- 001	- % 001		
899	011		
0-1 = 9; with borrow of 1 from next column 0 -1 (borrow) - $0 = 9$, with borrow of 1 9 - 1 (borrow) - $0 = 8$. Answer = 899.	0-1 = 1; with borrow of 1 from next column 0 - 1 (borrow) - $0 = 1$, with borrow of 1 1 - 1 (borrow) - $0 = 0$. Answer = % 011.		
	V 0.1	16	

Hex	Addition	
3Ah + 28h 62h A+8 = 2; with carry out of 1 to next column 1 (carry) + 3 + 2 = 6. answer = \$ 62.	Decimal check. $3Ah = 3 \times 16 + 10$ = 58 $28h = 2 \times 16 + 8$ = 40 58 + 40 = 98 $62h = 6 \times 16 + 2$ = 96 + 2 = 98!!	
	V 0.1	17

Hex addition again	
Why is $Ah + 8h = 2$ with a carry out of 1?	
The carry out has a weight equal to the BASE (in this case 16). The digit that gets left is the excess (BASE - sum).	
Ah + 8h = 10 + 8 = 18.	
18 is GREATER than 16 (BASE), so need a carry out!	
Excess is $18 - BASE = 18 - 16 = 2$, so '2' is digit.	
Exactly the same thing happens in Decimal. 5+7=2, carry of 1. 5+7=12, this is greater than 10!. So excess is $12 - 10 = 2$, carry of 1.	
V 0.1	18

Hex S	ubtraction	
34h - 27h 0Dh 4-7 = D; with borrow of 1 from next column 3 = 1 (borrow) = 2 = 0	Decimal check. $34h = 3 \times 16 + 4$ = 52 $27h = 2 \times 16 + 7$ = 39 52 - 39 = 13 0Dh = 13 !!	
3 - 1 (00100) - 2 - 0. answer = \$ 0D.	V 0.1	19



Fixed With paper and pencil, I can I want:	Precision write a number with as many digits as	
1,027,80,032,034,532,002	,391,030,300,209,399,302,992,092,920)
A microprocessor or comput PRECISION for integers; th number of bits:	ting system usually uses FIXED ney limit the numbers to a fixed	
\$ AF4500239DEFA231	64 bit number, 16 hex digits	
\$ 9DEFA231	32 bit number, 8 hex digits	
\$ A231	16 bit number, 4 hex digits	
\$ 31	8 bit number, 2 hex digits	
High end microprocessors microprocessors use 16 or	use 64 or 32 bit precision; low end 8 bit precision.	
	V 0.1 21	



Unsigned Overflow Example

Assume 8 bit precision; ie. I can't store any more than 8 bits for each number.

Lets add 255 + 1 = 256. The number 256 is OUTSIDE the range of 0 to 255! What happens during the addition? 255 = \$FF /= means Not Equal + 1 = \$ 01

-----256 /= \$00

F + 1 = 0, carry out F + 1 (carry) + 0 = 0, carry out Carry out of MSB falls off end, No place to put it !!! Final answer is WRONG because could not store carry out. V 0.1



Signed Integer Representation

We have been ignoring large sets of numbers so far; ie. the sets of signed integers, fractional numbers, and floating point numbers.

We will not talk about fractional number representation (10.3456) or floating point representation (i.e. 9.23×10^{13}).

We WILL talk about signed integer representation.

The *PROBLEM* with signed integers (-45, +27, -99) is the SIGN! How do we encode the sign?

The sign is an extra piece of information that has to be encoded in addition to the magnitude. Hmmmmm, what can we do??

V 0.1

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Twos Complement Examples

For 8 bits, can represent the signed integers -128 to +127.

For N bits, can represent the signed integers

 $-2^{(N-1)}$ to $+2^{(N-1)}-1$

Note that negative range extends one more than positive range.

Twos Complement Comments

Twos complement is the method of choice for representing signed integers.

There is only one zero, and K + (-K) = 0.

-5 + 5 =\$FB + \$05 = \$00 = 0 !!!

Normal binary addition is used for adding numbers that represent twos complement integers.

V 0.1



Example Conversions

\$FE as an 8 bit unsigned integer = 254 \$FE as an 8 bit twos complement integer = -2

\$7F as an 8 bit unsigned integer = 127 \$7f as an 8 bit twos complement integer = +127

To do hex to signed decimal conversion, we need to determine sign (Step 1), determine Magnitude (step 2), combine sign and magnitude (Step 3)

V 0.1







Hex to Signed Decimal (cont) STEP 3 : Just combine the sign and magnitude to get the result. \$F0 as 8 bit twos complement number is -16 \$64 as an 8 bit twos complement integer =+100











Some Examples				
All hex numbers represent sign format.	ed decimal in two's complement	ıt		
\$ FF = -1	\$ FF = -1			
+ \$ 01 = + 1	+ \$ 80 = -128			
00 = 0	F = +127 (incorrect!!)			
Note there is a carry out, but the answer is correct. Can't have 2's complement overflow when adding positive and negative	Added two negative numbers, got a positive number. Twos Complement overflow.			
number.	V 0.1	39		







Codes for Characters

Also need to represent Characters as digital data. The ASCII code (American Standard Code for Information Interchange) is a 7-bit code for Character data. Typically 8 bits are actually used with the 8th bit being zero or used for error detection (parity checking). 8 bits = 1 Byte. (see Table 2.5, pg 47, Uffenbeck).

'A' = % 01000001 = \$41'&' = % 00100110 = \$26

7 bits can only represent 27 different values (128). This enough to represent the Latin alphabet (A-Z, a-z, 0-9, punctuation marks, some symbols like \$), but what about other symbols or other languages? V 0.1

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		lunuu	u 00u	0 101 1	nonne		leiche	inge
Table 2.5 A	merican St	andard Cod	le for Infon	mation Inte	rchange (A	SCII)*		
Land				Most Sign	ificant Bit			
Significant Bit	0 0000	1 0001	2 0010	3 0011	4	5 0101	6 0110	0111
0.0000	NUL	DLE	SP	0	0	Р	18	p
1 0001	SOH	DCI	1	1	A	Q		9
2 0010	STX	DC2		2	B	R	b	r
3 0011	ETX	DC3		3	C	8	•	
4 0100	EOT	DC4	5	4	D	т	d	
5 0101	ENQ	NAK	16	5	E	U	¢	
6 0110	ACK	SYN	80	6	F	v	r	v
7 0111	BEL.	ETB		7	G	w	8	w
8 1000	BS	CAN	(8	н	x	h	x
9 1001	HT	EM)	9	1	Y	1 A -	У
A 1010	LF	SUB		:	1	Z	3	z
B 1011	VT	ESC	+		K	- T	k	1
C 1100	FF	FS		~	L.	1	1	1
D 1101	CR	GS	-		M	1	m	1
E 1110	SO	RS		>	N	^	n	
F 1111	SI	US	/	2	0		0	DEL

UNICODE UNICODE is a 16-bit code for representing alphanumeric data. With 16 bits, can represent 2^{16} or 65536 different symbols. 16 bits = 2 Bytes per character.				
\$0041-005A A-Z \$0061-4007A a-z				
Some other alphabet/symbol ranges				
\$3400-3d2dKorean Hangul Symbols\$3040-318FHiranga, Katakana, Bopomofo, Hangul\$4E00-9FFFHan (Chinese, Japenese, Korean)				
UNICODE used by Web browsers, Java, most software these days. $$v_{0.1}$$				

Codes for Decimal Digits					
There are even codes for representing decimal digits. These codes use 4-bits for EACH decimal digits; it is NOT the same as converting from decimal to binary.					
BCD Code 0 = % 0000 1 = % 0001 2 = % 0010	In BCD code, each decimal digit simply represented by its binary equivalent. 96 = % 1001 0110 = \$ 96 (BCD code)				
3 = % 0011 4 = % 0100	Advantage: easy to convert Disadvantage: takes more bits to store a number:				
5 = % 0101 6 = % 0110 7 = % 0111	255 = % 1111 1111 = \$ FF (binary code) 255 = % 0010 0101 0101 = \$ 255 (BCD code)				
8 = % 1000 9 = % 1001	takes only 8 bits in binary, takes 12 bits in BCD. $$_{\rm V0.1}$$				

What do you need to Know? · Convert hex, binary integers to Decimal · Convert decimal integers to hex, binary • Convert hex to binary, binary to Hex N binary digits can represent 2^N values, unsigned • integers 0 to 2^{N-1} . Addition, subtraction of binary, hex numbers ٠ · Detecting unsigned overflow Converting a decimal number to Twos • Complement Converting a hex number in 2s complement to • decimal V 0.1 48

Gray Code for decimal Digits

$\begin{array}{rcl} \text{Gray Code} \\ 0 &= \ \% \ 0000 \\ 1 &= \ \% \ 0001 \\ 2 &= \ \% \ 0011 \\ 3 &= \ \% \ 0010 \\ 4 &= \ \% \ 0110 \\ 5 &= \ \% \ 1010 \\ 6 &= \ \% \ 1011 \\ 8 &= \ \% \ 1001 \\ 9 &= \ \% \ 1000 \end{array}$	A Gray code changes by only 1 bit for adjacent values. This is also called a 'thumbwheel' code because a thumbwheel for choosing a decimal digit can only change to an adjacent value (4 to 5 to 6, etc) with each click of the thumbwheel. This allows the binary output of the thumbwheel to only change one bit at a time; this can help reduce circuit complexity and also reduce signal noise.
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V 0.1

What do you need to know? (cont)

- Number ranges for 2s complement
- Overflow in 2s complement
- Sign extension in 2s complement
- ASCII, UNICODE are binary codes for character data
- BCD code is alternate code for representing decimal digits
- Gray codes can also represent decimal digits; adjacent values in Gray codes change only by one bit.

V 0.1